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The Response of the Tropical Atmosphere
to Low Frequency Thermal Forcing

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Abstract

The tropical response to a localized thermal forcing with approximately 45 day period is investigated using several models which consist of two equivalent shallow water systems and a fully stratified system. The fully stratified model appears to be able to reproduce a number of features of the tropical 40-50 day oscillation including the observed eastward and poleward propagation of zonal wind anomalies.

1. Introduction

The goal of this work is to determine the structure of the response of a tropical atmosphere to slow oscillatory forcing with period of approximately 45 days. Our principal motivation for studying this problem is to determine whether the structure of the observed 40-50 tropical day oscillation which was first reported by Madden and Julian (1971) is consistent with the response expected from a localized thermal forcing. Observational studies by Yasunari (1980) and Julian and Madden (1981) show that there is a modulation of convective rainfall which presumably provides such forcing with 40-50 day period in the west Pacific/Indonesian region.

We explore the tropical response using a number of successively more complicated models so that we can identify which dynamical effects are critically important. The first model which we use is a simple linear shallow water system. This system is obtained from the fully stratified atmospheric problem by performing a linearization about a resting basic state (Lindzen, 1967; Gill, 1980). This calculation is essentially identical to that of Yamagata and Hayashi (1984) and represents the simplest possible approach to the forced problem. The second model which we study is linearized about a zonally symmetric basic state consisting of pure zonal flow. This is the most general basic state for which a shallow water-like system can be derived. Finally we employ a full 3-dimensional model with a basic state consisting of a zonally symmetric Hadley cell which includes the effects of momentum transport by the cumulus clouds and the advection of perturbation quantities by the mean vertical and meridional winds.

Recent modeling studies (Anderson, 1984a; Anderson 1984b) have shown that the inclusion of the Hadley circulation in the basic state results in the formation of zonally symmetric model eigenmodes which represent damped resonances in the 40-50 day period range. In Section 3 we will study the effects of the inclusion of the dynamics responsible for these modes on the solution to the forced problem.

2. Equivalent shallow water model calculations

The derivation of the model which Yamagata and Hayashi (1984) used follows closely the model developed by Gill (1980) to study the tropical response to steady state forcing. It is based on the separation of the problem in the vertical and horizontal directions. The forcing is then projected onto one eigenmode of the resulting vertical structure problem and the evolution of the system can be studied by solving a set of shallow-water like equations. The most general basic state for which this separation can be accomplished is given by $\bar{u}=\bar{u}(y)$, $\bar{v}=0$, $\bar{w}=0$, $\partial\bar{\theta}/\partial y = 0$ (cf. Stevens, 1983) and yields the following system for the horizontal structure problem;

$$\frac{\partial u}{\partial t} + \bar{u} \frac{\partial u}{\partial x} - fv = - \frac{\partial \phi}{\partial x} - \alpha u \quad (1.a)$$

$$\frac{\partial v}{\partial t} + \bar{u} \frac{\partial v}{\partial x} + fu = - \frac{\partial \phi}{\partial y} - \alpha v \quad (1.b)$$

$$\frac{\partial \phi}{\partial t} + \bar{u} \frac{\partial \phi}{\partial x} = - gH \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + Q - \alpha \phi. \quad (1.c)$$

Here $u(x,y,t)$, $v(x,y,t)$ and $\phi(x,y,t)$ are the horizontal structure

variables defined by $u(x,y,z,t) = u(x,y,t) \cdot \hat{u}(z) \dots$, where $\hat{u}(z)$, $\hat{v}(z)$, and $\hat{\phi}(z)$ are the eigenfunctions of the vertical structure problem. H is the equivalent depth which is determined from the eigenvalue of the mode which the forcing is taken to be projected onto, $Q(x,y,t)$ is the amplitude of the forcing and α is a simple Rayleigh friction/ Newtonian cooling representation for the dissipation. With this choice of heating sign the wind field corresponds to upper tropospheric winds for the original stratified fluid.

Yamagata and Hayashi made an additional approximation to the above system known as the "long-wave" approximation (Gill, 1980) which consists of ignoring the terms involving v in (1.b); however, as will be evident, this makes no noticeable difference in the solution. Since the system is constant coefficient in the x -direction, it is traditional to take a discrete Fourier series representation in that direction, yielding

$$\frac{\partial u^k}{\partial t} + \bar{u} \cdot iku^k - fv^k = -iku^k - \alpha u^k \quad (2.a)$$

$$\frac{\partial v^k}{\partial t} + \bar{u} \cdot ikv^k + fu^k = -\frac{\partial \phi^k}{\partial y} - \alpha v^k \quad (2.b)$$

$$\frac{\partial \phi^k}{\partial t} + \bar{u} \cdot ik\phi^k = -gH(iku^k + \frac{\partial v^k}{\partial y}) + Q^k - \alpha \phi^k, \quad (2.c)$$

where $a(x,y,t) = \sum_k a^k(y,t) \exp(ikx)$.

Now if we use a suitable discretization in the y direction we can write the system in schematic form as

$$\frac{\partial \underline{S}^k}{\partial t} = \underline{A}^k \underline{S}^k + \underline{Q}^k$$

where the vector \underline{S}^k is the system state vector (u, v, ϕ) for wavenumber k and the matrix \underline{A}^k represents the linear dynamics operator. The discretization for all of our calculations was accomplished using a Fourier representation in the meridional direction; \underline{A} is calculated using transform techniques. In order to mask the intrinsic periodicity of the basis functions, an absorbing layer at the poleward limits of the domain was incorporated into α . If the system is stable (real part of all eigenvalues of $\underline{A} < 0$) and the forcing $\underline{Q}^k \alpha \exp(i\omega t)$ then we can represent the forced system by the matrix problem

$$i\omega \underline{R}^k = \underline{A}^k \underline{R}^k + \underline{Q}^k$$

or $(i\omega \underline{I} - \underline{A}^k) \underline{R}^k = \underline{Q}^k,$ (3)

where \underline{R}^k is the system response vector for zonal wavenumber k . The complete response is then determined by solving (3) to get $\underline{R}(x,y)$ for each zonal wavenumber component of the forcing and summing the results.

Yamagata and Hayashi used the parameters previously determined by Gill, which are listed below, and made the forcing oscillatory in time.

$$Q = \begin{cases} \exp(i\omega t) \cdot \exp[-(\frac{y}{L_y})^2] \cdot \cos(k_x x) & \text{for } |k_x x| < \pi \\ 0 & \text{for } |k_x x| > \pi \end{cases} \quad (4)$$

$$H = 400 \text{ m}$$

$$L_y = 2.34 \cdot 10^6 \text{ m}$$

$$k_x = \pi / (9.4 \cdot 10^6 \text{ m})$$

$$\alpha = 1 / (2.2 \text{ days}) \quad \text{away from absorbing layers}$$

$$\bar{u} = 0$$

$$\omega = 2\pi / (45 \text{ days})$$

A plot of the heating field is given in Figure 1. It should be noted that the y-extent of the heating is rather large and the damping time is very fast, particularly for the subtropics; these points will be addressed later. The result of solving (3) using the Yamagata and Hayashi parameters on a domain extending from 60°S to 60°N with 30 meridional wavenumbers and zonal wavenumbers ranging from -10 to 10 are shown in Figure 2. These results are essentially identical to those found by Yamagata and Hayashi who employed the "long-wave" approximation on an infinite β -plane without discretization in the y-direction; the favorable comparison verifies our use of the Fourier basis functions. Notice that for each variable the response at $\omega t=0$ is large compared to the propagating response depicted by $\omega t=90^{\circ}$. This result is due to the damping time being significantly shorter than the oscillatory time scale ($\alpha \gg \omega$) (cf. Geisler and Stevens, 1982). In fact the results are nearly identical to those obtained by Gill for $\omega=0$.

The second case which we consider is an attempt to make the most realistic calculation possible within the constraints of a separable (equivalent shallow water) model. Here we have reduced the value of the model dissipation ($\alpha = 1/(12 \text{ days})$) and we have adapted a smaller meridional extent for the heating ($L_y = 1.25 \cdot 10^6 \text{m.}$) The heating field is shown in Figure 3a. The meridional scale was chosen in an attempt to represent the observed fluctuations in the Pacific convection. While the dissipative time scale for a model this simple is always somewhat arbitrary, we have chosen a less dissipative system so that a propagating response is at least possible.

The basic state zonal wind field, $\bar{u}(y)$, is shown in Figure 3 and is taken from a mass weighted vertical average of the two dimensional field which we use in Section 3. The forced problem was solved using the same technique as above on a domain extending from 43°S to 43°N and the results appear as Figure 4. The in-phase ($wt = 0$) fields show a strong similarity to the Yamagata and Hayashi results while the $wt = 90^\circ$ response is now of comparable magnitude and indicates a general westward propagation of disturbance. This feature is not in good agreement with the observed eastward propagation of the oscillation as described by Madden and Julian (1972). In the next section we will see if the inclusion of other physical processes can improve the agreement with observations.

3. Calculations with a Hadley cell basic state

In this section we will now expand the basic state definition for our linear problem to include a general two-dimensional (latitude-height) description. We have chosen to limit this calculation to the effects of a zonally symmetric basic state due to significant computational difficulties associated with allowing the basic state variables to vary in the third spatial dimension. In effect we are including the new dynamics introduced by the "Hadley cell" but have not yet included a "Walker" type circulation in our model basic state. As part of an ongoing research effort we are currently examining various approaches to relax this restriction.

One of the major advantages of eliminating the requirement for vertical separability is that we can now include a more physical representation of internal dissipation than the Rayleigh friction/ Newtonian

cooling formulations. The approach which we will adopt here is the so-called "cumulus friction" introduced by Schneider and Lindzen (1976) where the dissipation results from vertical momentum transport by cumulus clouds. An important aspect of this form is that the dissipation is concentrated in regions of basic state heating.

The linear perturbation equations describing hydrostatic flow on an equatorial β -plane about this basic state are given for each zonal wavenumber k by:

$$\frac{Du'}{Dt} = \beta y v' - \frac{1}{\tilde{\rho}} p'_x - \alpha(y, z) u' + \mu u'_{zz} + F_{cx} \quad (5a)$$

$$\frac{Dv'}{Dt} = -\beta y u' - \frac{1}{\tilde{\rho}} p'_y - \alpha(y, z) v' + \mu v'_{zz} + F_{cy} \quad (5b)$$

$$\frac{D\theta'}{Dt} = Q' - \gamma(y, z) \theta' + \mu \theta'_{zz} \quad (5c)$$

$$-\frac{1}{\tilde{\rho}} (\tilde{\rho} w')_z = u'_x + v'_y \quad (5d)$$

$$\left(\frac{p'}{\tilde{\rho}} \right)_z = \frac{g}{\tilde{\theta}} \theta' \quad (5e)$$

$$\frac{D(\)'}{Dt} = (\)'_t + u'(\bar{\ })_x + v'(\bar{\ })_y + w'(\bar{\ })_z + \bar{u}(\)'_x + \bar{v}(\)'_y + \bar{w}(\)'_z \quad (5f)$$

where $\frac{\partial}{\partial x} = ik$ and $\tilde{\rho}(z)$ and $\tilde{\theta}(z)$ are a hydrostatically balanced reference stratification. F_{cx} and F_{cy} are the cumulus momentum transport terms given by:

$$F_{cx} = \frac{1}{\tilde{\rho}} \left[(\bar{M}_c(u' - u'_c))_z + (M'_c(\bar{u} - \bar{u}_c))_z \right] \quad (6a)$$

$$F_{cy} = \frac{1}{\tilde{\rho}} \left[(\bar{M}_c(v' - v'_c))_z + (M'_c(\bar{v} - \bar{v}_c))_z \right] \quad (6b)$$

the cloud mass transport M_c is given by:

$$\bar{M}_c(y, z) = M_c(y) \left\{ 1 - \left[\exp \frac{p_T - p}{p_{DTR}} \right] \right\} \quad p_T \leq \tilde{p} \leq p_c$$

$$p_T \sim 150 \text{ mb} \quad p_{DTR} = 75 \text{ mb} \quad p_c \sim 900 \text{ mb}$$

Here the basic state is given by \bar{u} , \bar{v} , \bar{w} , $\bar{\theta}$ and \bar{M}_c which is the cumulus mass flux as defined by Schneider and Lindzen. The basic state fields which are presented in Figure 5 are found by integrating a non-linear version of (5) with a specified heating field, $\bar{Q}(y, z)$. \bar{Q} and Q' are related to $\bar{M}_c(y)$ and $M'_c(y)$ by an integral constraint which requires the total heat released to be equal to the cloud moisture flux in a fashion similar to the cumulus parameterization scheme proposed by Stevens and Lindzen (1978).

In the context of our more complete model we will now use the simple dissipation parameters α and γ solely for the purpose of imposing boundary conditions. The Rayleigh friction terms are used to impose the poleward boundary sponge and surface frictional effects while the Newtonian cooling term is used to absorb energy at the top of the domain and simulate a radiation condition.

The horizontal discretization is accomplished as before using Fourier basis functions; a staggered centered difference formulation is used for vertical derivatives. A small vertical viscosity term is used to smooth the effects of the vertical differencing scheme. All of the results presented in this section were calculated with a 20 meridional wavenumber truncation on a 43°N-43°S domain and 20 vertical levels from 1 to 18 km. The three dimensional fields were constructed by summing the results of zonal wavenumbers 0-4, which was found to give good

convergence for our forcing. In principle the system could be solved using linear algebra techniques as in the separable case; however in practice, the large order of the matrix (1200 x 1200 complex elements) makes this approach computationally intractable. Instead each wavenumber is solved by integrating a prognostic model with oscillating forcing until an equilibrium oscillating response is reached. On a Cray-1 computer the solution requires approximately 200 seconds of CPU time for each wavenumber using semi-implicit time integration techniques.

The forcing function for this case was chosen to have the same horizontal structure as the second shallow water case with a vertical distribution which is given by a Gaussian centered at 400 mb and an e/folding distance of 5.1 km. Since the calculation is linear the actual magnitude of the heating is arbitrary; however the response fields are presented so that they are consistent with a 1°C/day heating maximum.

The response u-component wind field for the model 200 mb level is presented in Figure 6. This figure shows a number of new features which were not present in the shallow water results. One striking feature is the presence of a strong response maximum located to the east of the heating region at approximately 20° latitude. The presence of this feature is in good agreement with observations and will be discussed later in this section in the context of the wavenumber 0 response. A second new result is the presence of an eastward propagating feature on the equator. This feature resembles the observations by Madden and Julian (1972) and may be due to the excitation of the viscous Kelvin wave mechanism proposed by Chang (1977).

The 830 mb response is shown in Figure 7 and shows no noticeable tendency toward eastward propagation. This is not in good agreement with observations and may result from the fact that the forcing in our calculations oscillates in time but remains localized at one longitude, whereas the observed cloudiness maximum and presumably the heating propagate eastward.

In order to examine the subtropical response in some detail we have displayed latitude-height cross sections of the zonal wavenumber 0 u-wind response in Figure 8. This figure shows a well defined maximum which originates near the equator and then propagates poleward in the upper troposphere. This is in good agreement with the observed zonally averaged field presented by Anderson and Rosen (1983) and the partial zonal average presented by Murakami et al. (1983) which are reproduced as Figures 9 and 10. Based on these figures, it appears that the model result may be in somewhat better agreement with the Murakami et al. fields, which are averaged over only the monsoon region, in the sense that the model displays upward propagation on the equator. It is possible that this is a result of our assumption of a zonally invariant basic state. We will pursue this topic in future research.

4. Summary

In conclusion we have found that the linear response of the tropical atmosphere to low frequency thermal forcing for the case where the basic state is a zonally symmetric Hadley cell produces results which differ considerably from calculations based on equivalent shallow water systems. The fully stratified results which include the Hadley cell effects appear to be in better agreement with observations

of the tropical 40-50 day oscillation in that they reproduce upper tropospheric motions with slow eastward and poleward phase propagation.

The presence of the poleward propagating modes is probably associated with the oscillating eigenmodes described in Anderson (1984a) and Anderson (1984b), which are also the subject of a paper now in preparation, while the eastward propagating equatorial feature may be associated with the viscous Kelvin wave mechanism proposed by Chang (1977).

It is our hope that this work will help lead to a better understanding of the processes which are important for the tropical 40-50 day oscillation and its various manifestations in the global atmosphere.

Acknowledgements

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Figure Legends

- Fig. 1 Forcing function (geopotential meters/day) for reproduction of Yamagata and Hayashi (1984) calculation.
- Fig. 2a u-component wind response (m/s) to Fig. 1 forcing.
- Fig. 2b v-component wind response (m/s) to Fig. 1 forcing.
- Fig. 2c Geopotential height response (gpm) to Fig. 1 forcing.
- Fig. 3a Forcing function for the general shallow water calculation.
- Fig. 3b $\bar{u}(y)$ field for the general shallow water calculation.
- Fig. 4a u-component wind response (m/s) for the general shallow water calculation.
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- Fig. 5a Heating field (deg/day) for calculation of Hadley cell basic state.
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- Fig. 5e Potential temperature perturbation (deg) for Hadley cell basic state.

- Fig. 6 U-component wind response (m/s) at 200 mb for fully stratified, Hadley cell basic state calculation.
- Fig. 7 U-component wind response (m/s) at 830 mb for fully stratified, Hadley cell basic state calculation.
- Fig. 8a Wavenumber 0 response at $\omega t = -45$ deg for Hadley cell calculation.
- Fig. 8b Wavenumber 0 response at $\omega t = 0$ for Hadley cell calculation.
- Fig. 8c Wavenumber 0 response at $\omega t = 45$ deg for Hadley cell calculation.
- Fig. 8d Wavenumber 0 response at $\omega t = 90$ deg for Hadley cell calculation.
- Fig. 9 (a)Amplitude and (b) phase (rad/ π) plot of the observed oscillation, from Anderson & Rosen (1983).
- Fig. 10 Latitude-height structure of partial zonal average (60°E to 150°E) u-wind field (with interval 1 ms^{-1}) at 45° phase intervals. From Murakami et al. (1983).

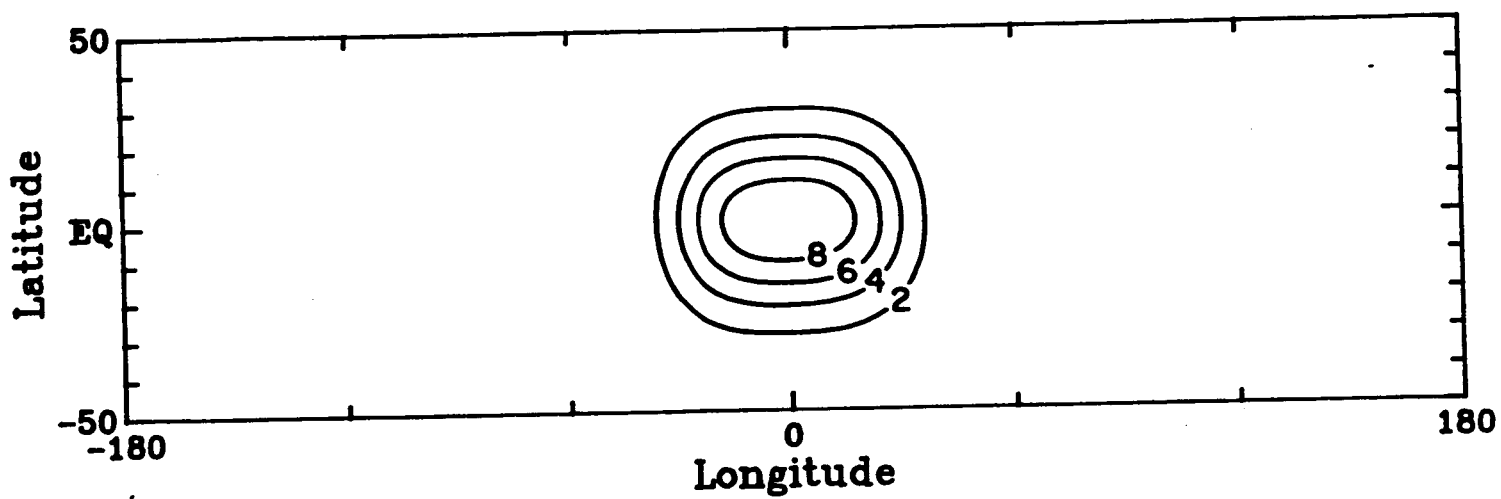


Fig. 1 Forcing function (geopotential meters/day) for reproduction of Yamagata and Hayashi (1984) calculation.

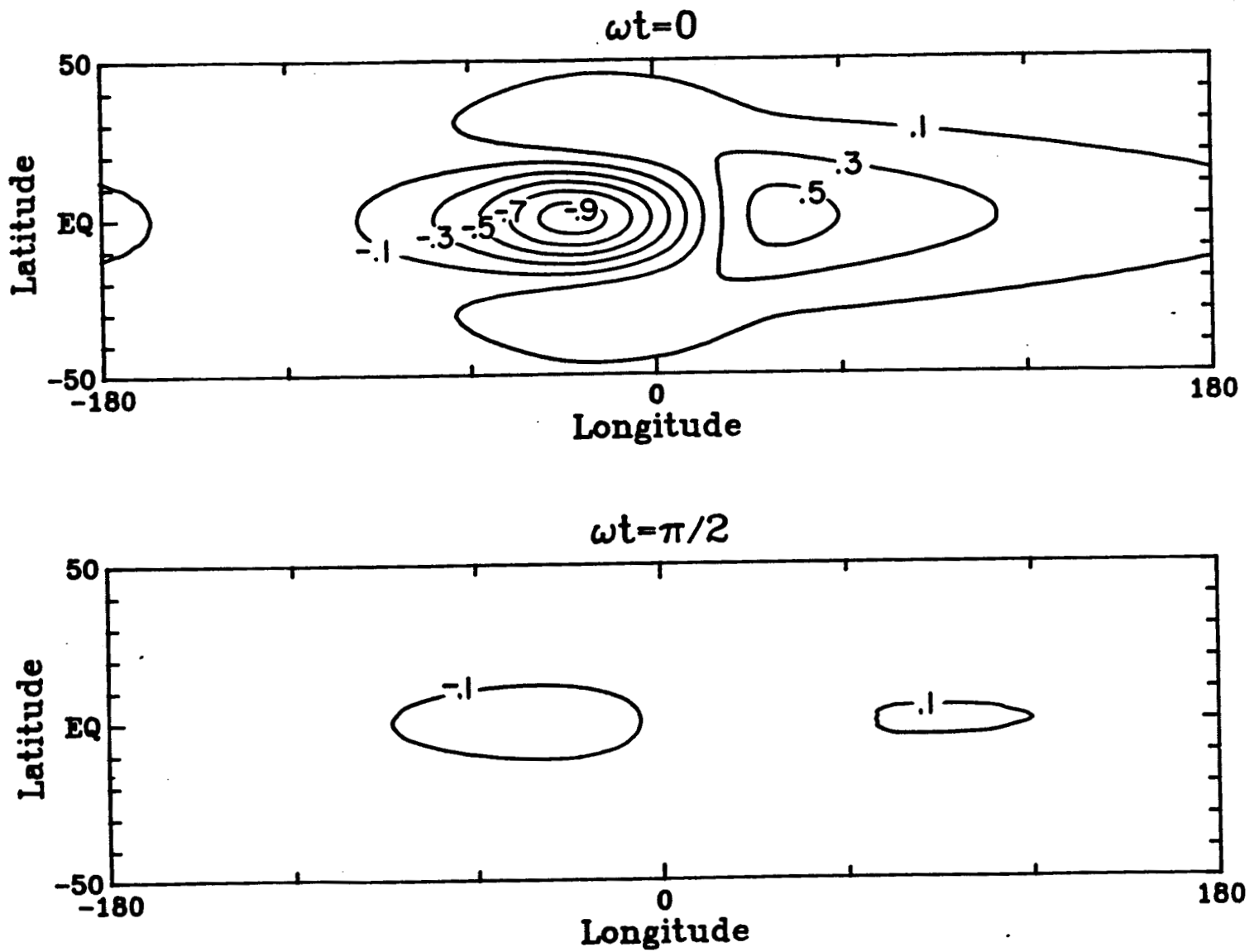


Fig. 2a u-component wind response (m/s) to Fig. 1 forcing.

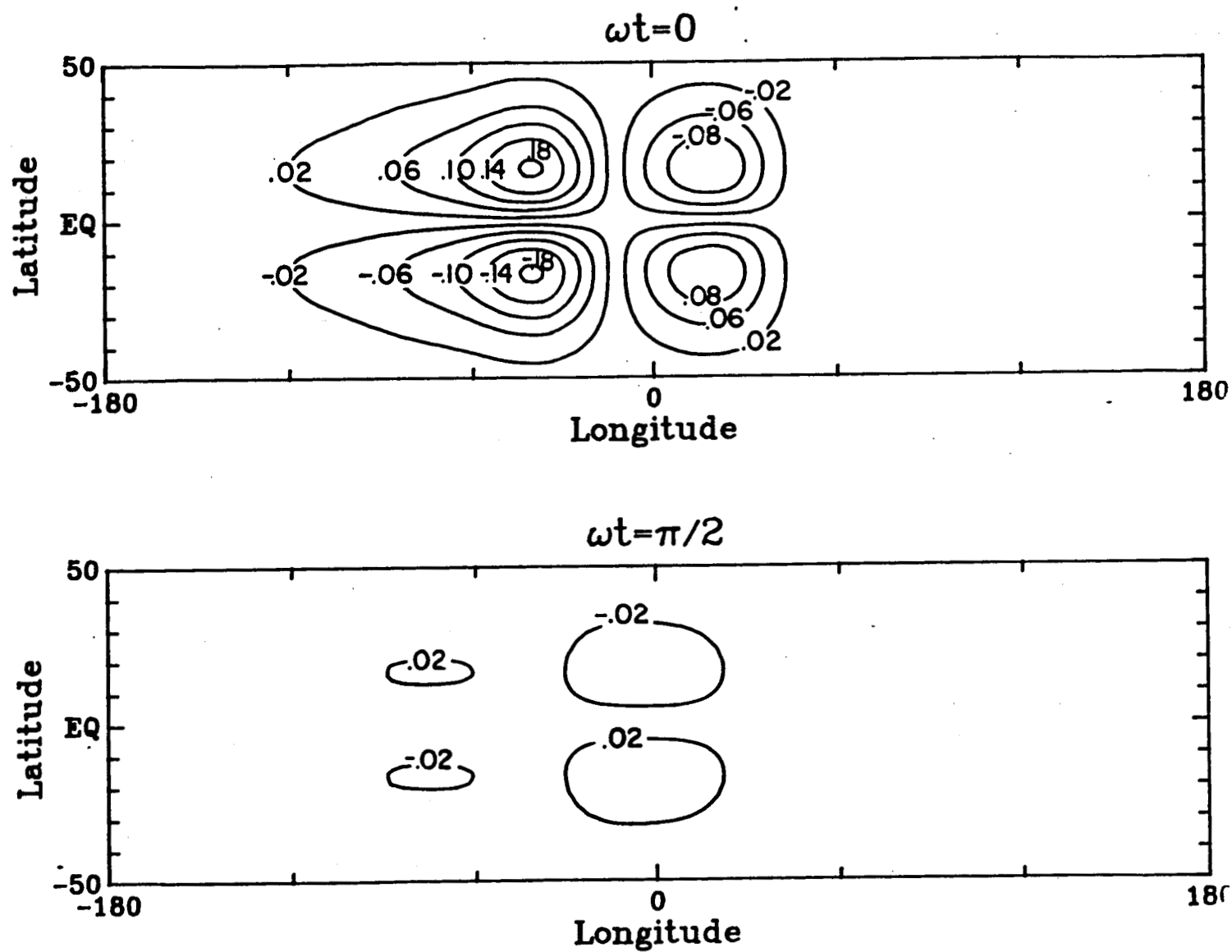


Fig. 2b v-component wind response (m/s) to Fig. 1 forcing.

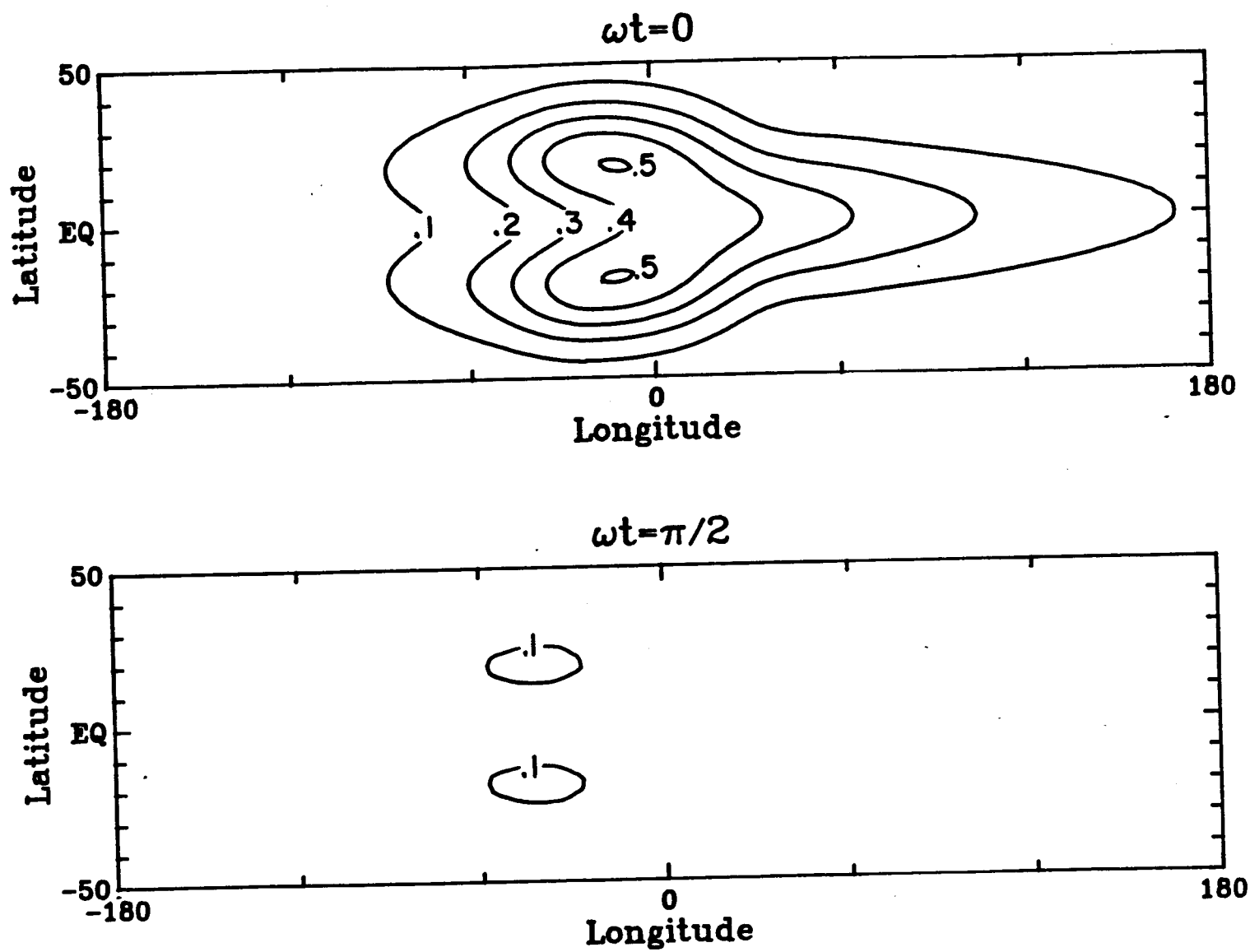


Fig. 2c Geopotential height response (gpm) to Fig. 1 forcing.

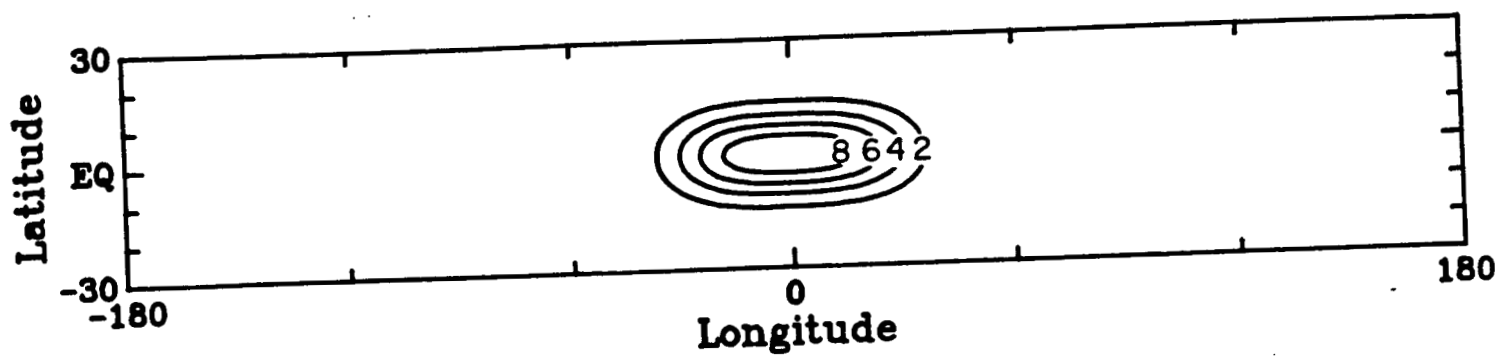


Fig. 3a Forcing function for the general shallow water calculation.

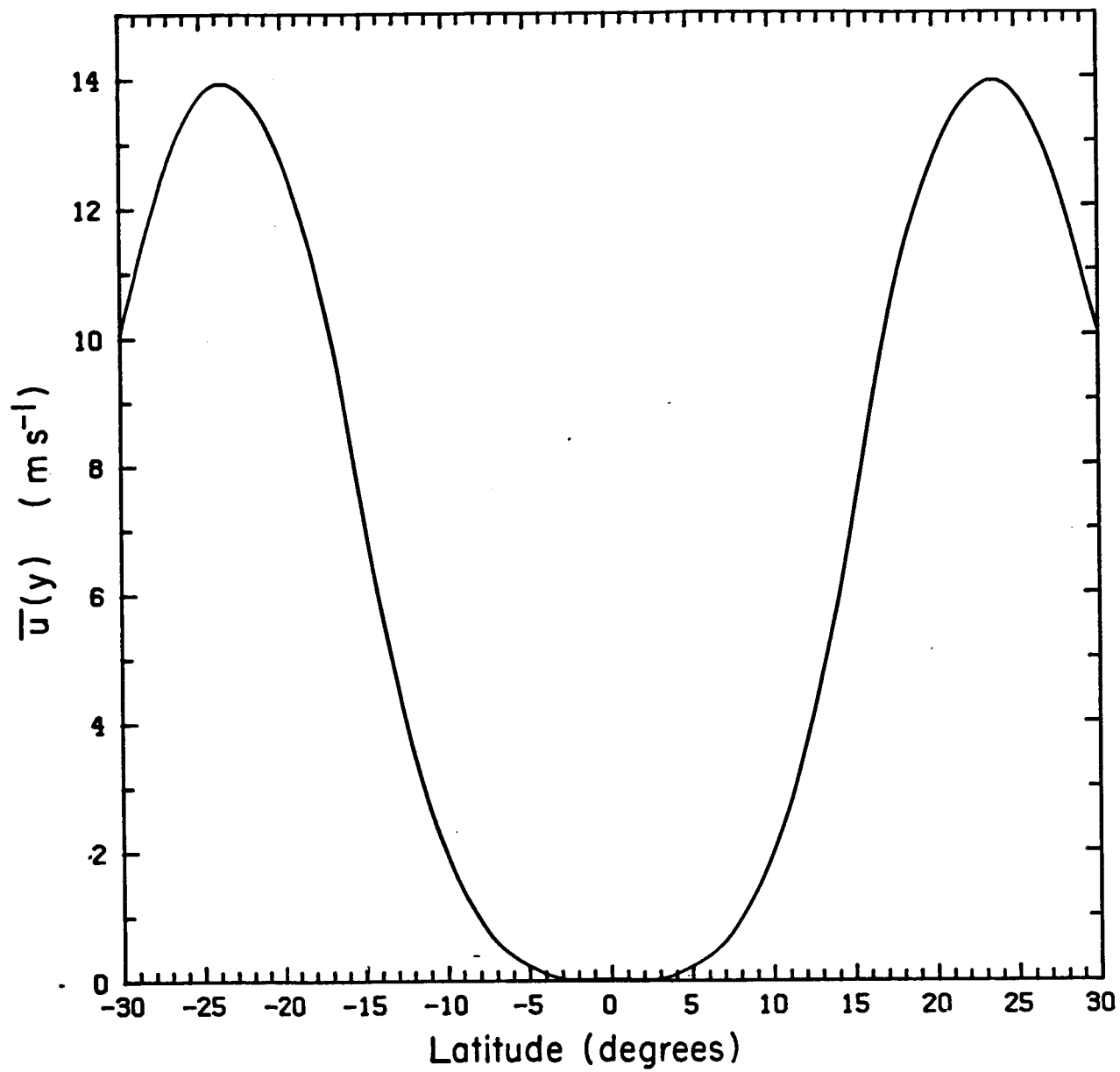


Fig. 3b $\bar{u}(y)$ field for the general shallow water calculation.

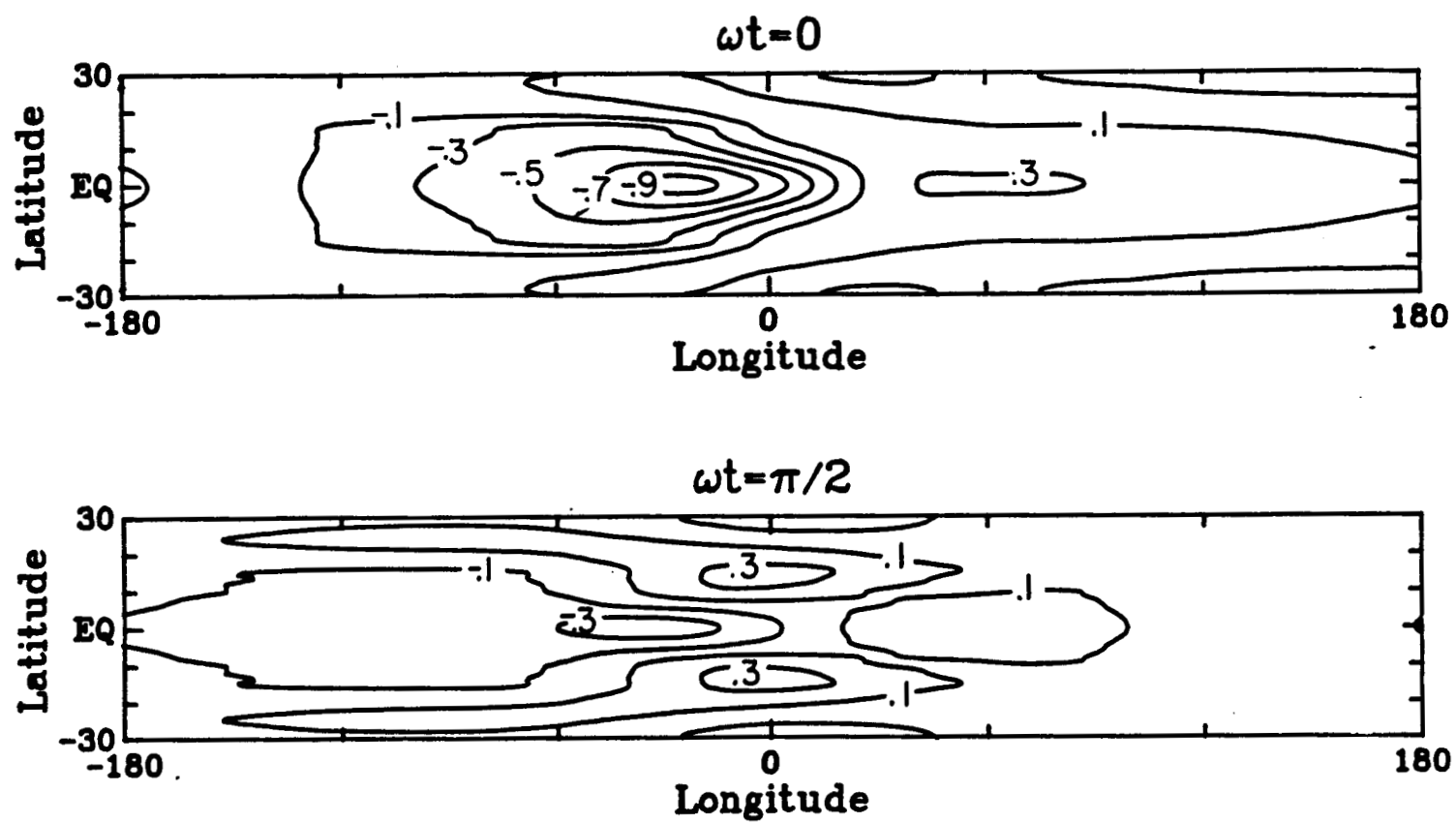


Fig. 4a u-component wind response (m/s) for the general shallow water calculation.

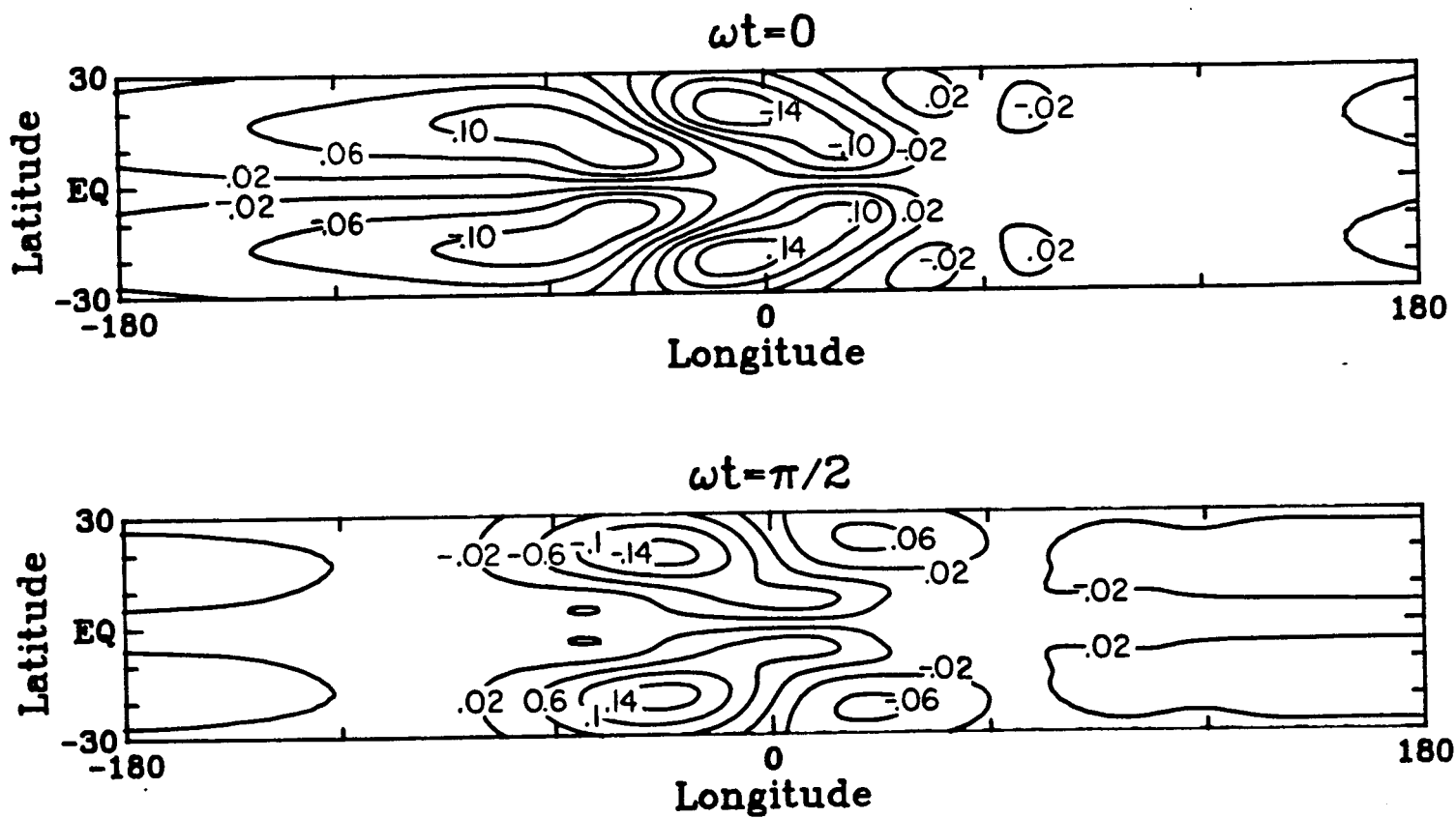


Fig. 4b v-component wind response (m/s) for the general shallow water calculation

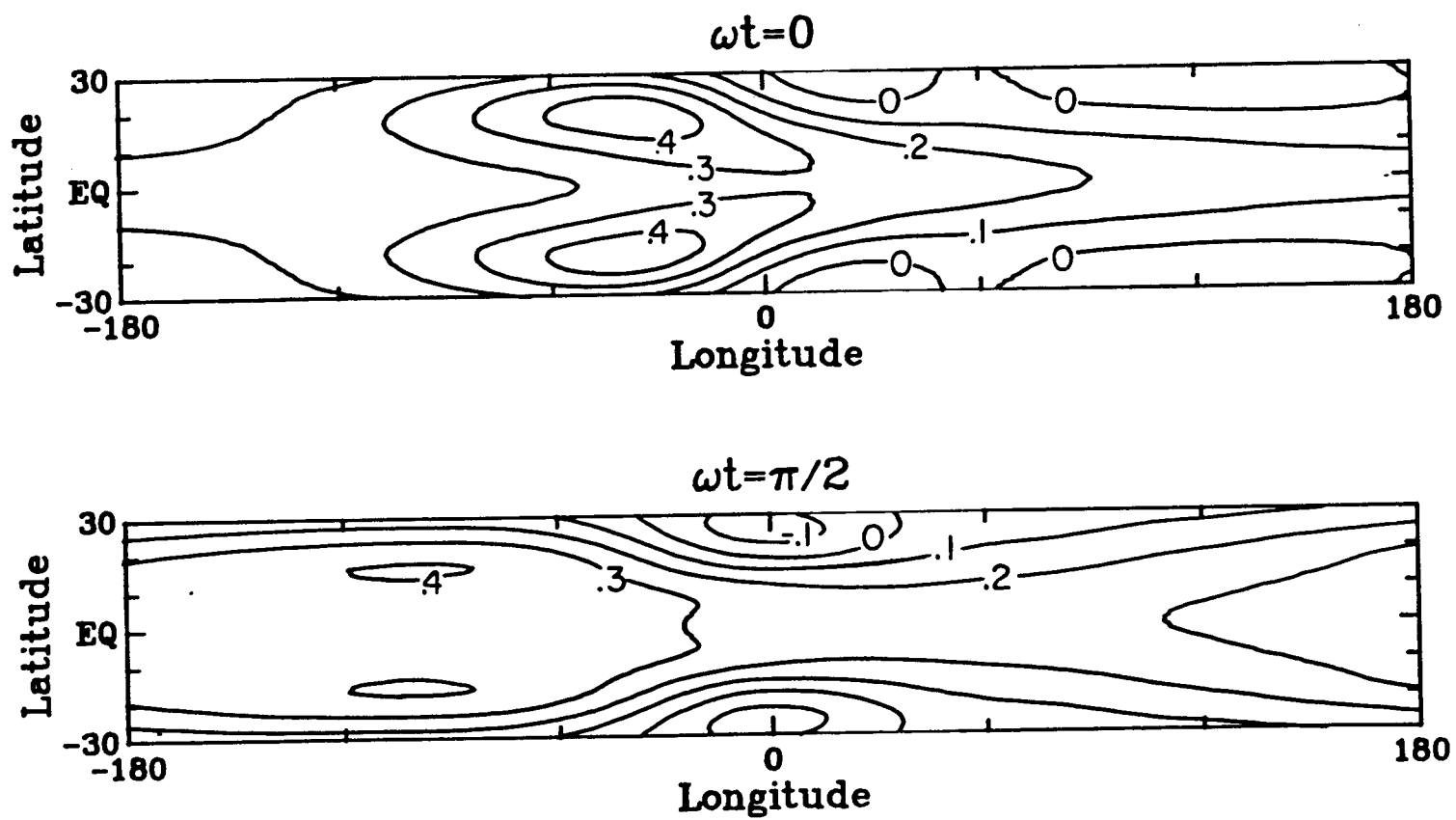


Fig. 4c Geopotential height response (gpm) for the general shallow water calculation.

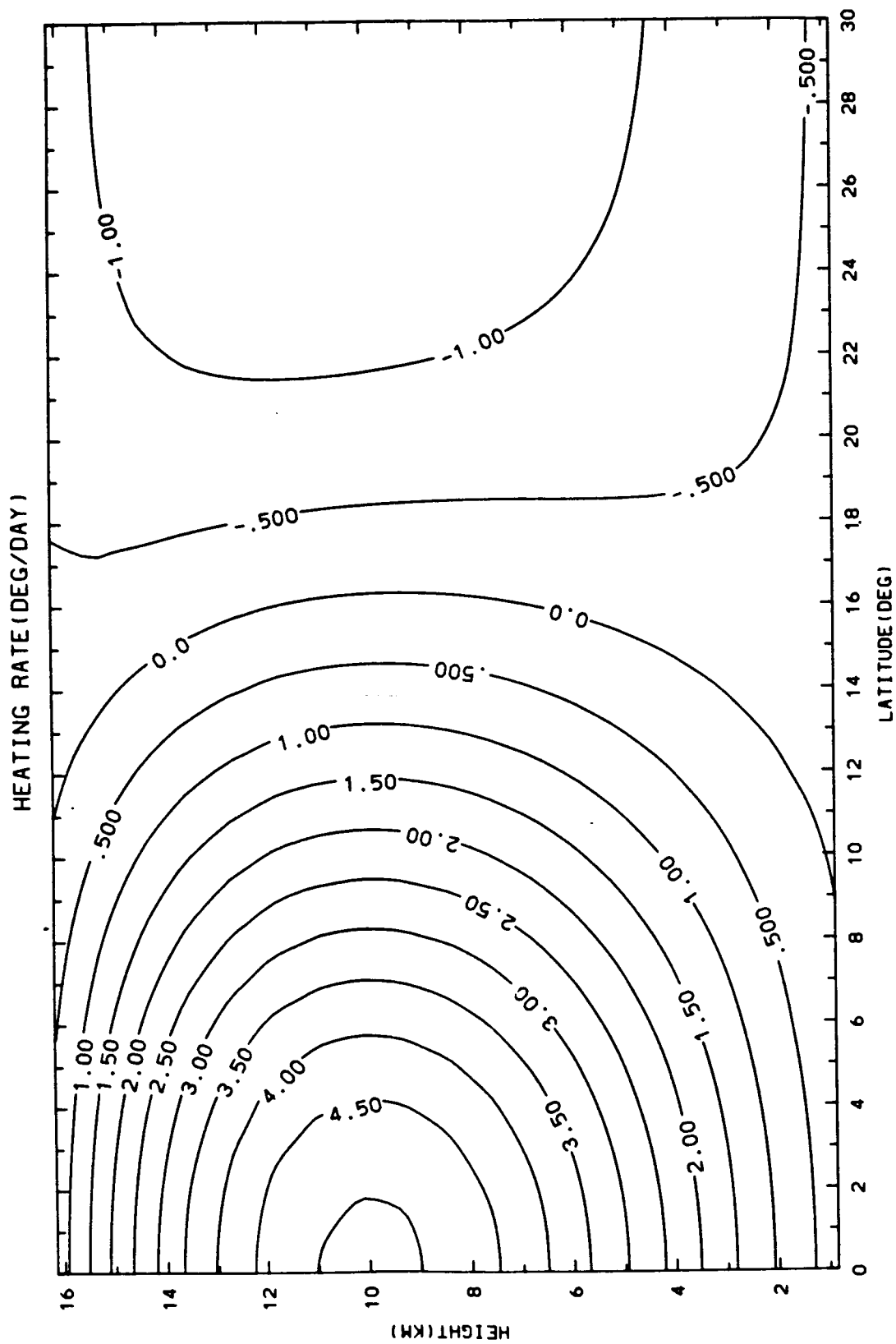


Fig. 5a Heating field (deg/day) for calculation of Hadley cell basic state.

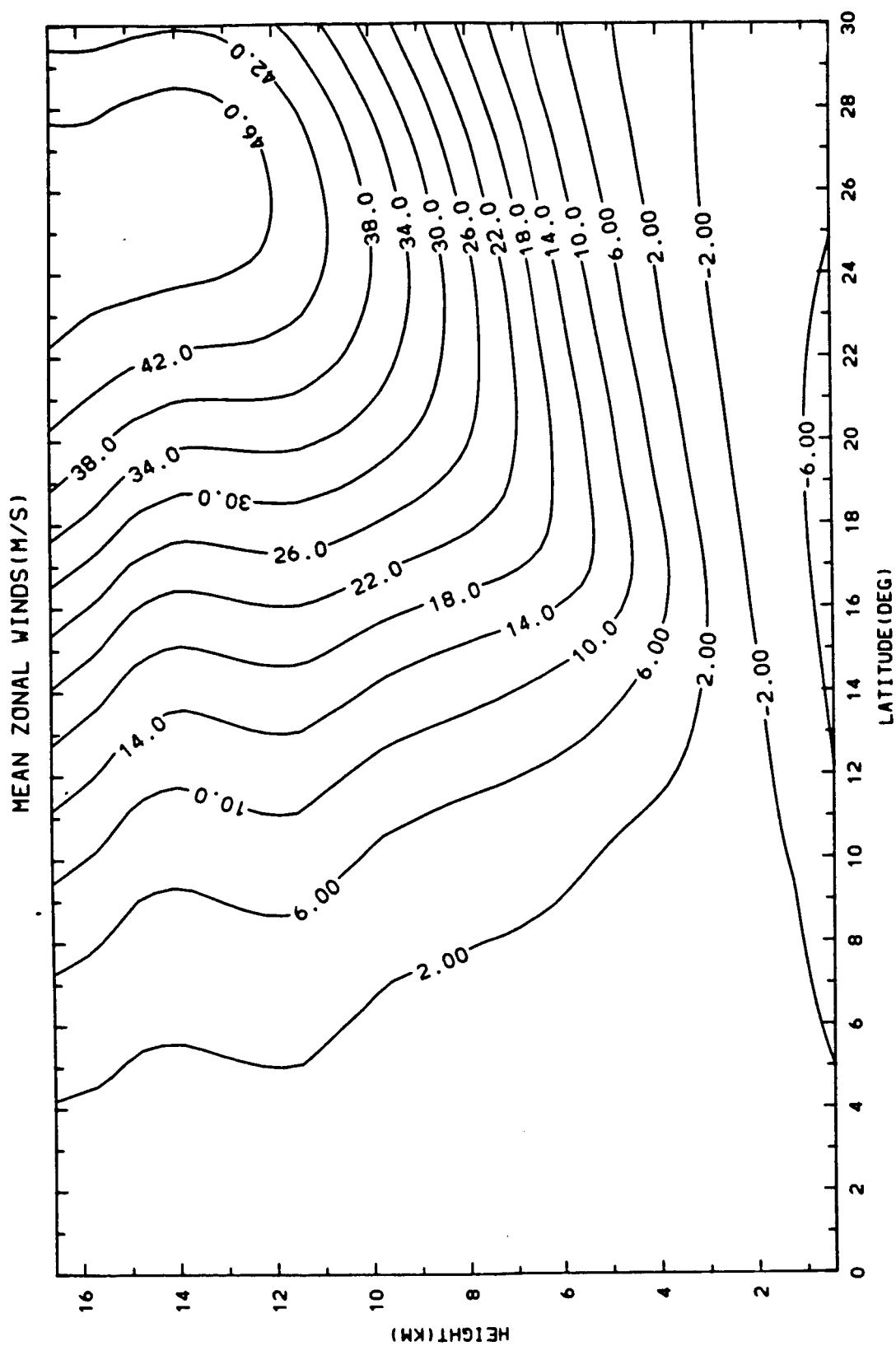


Fig. 5b u-component wind field (m/s) of Hadley cell basic state.

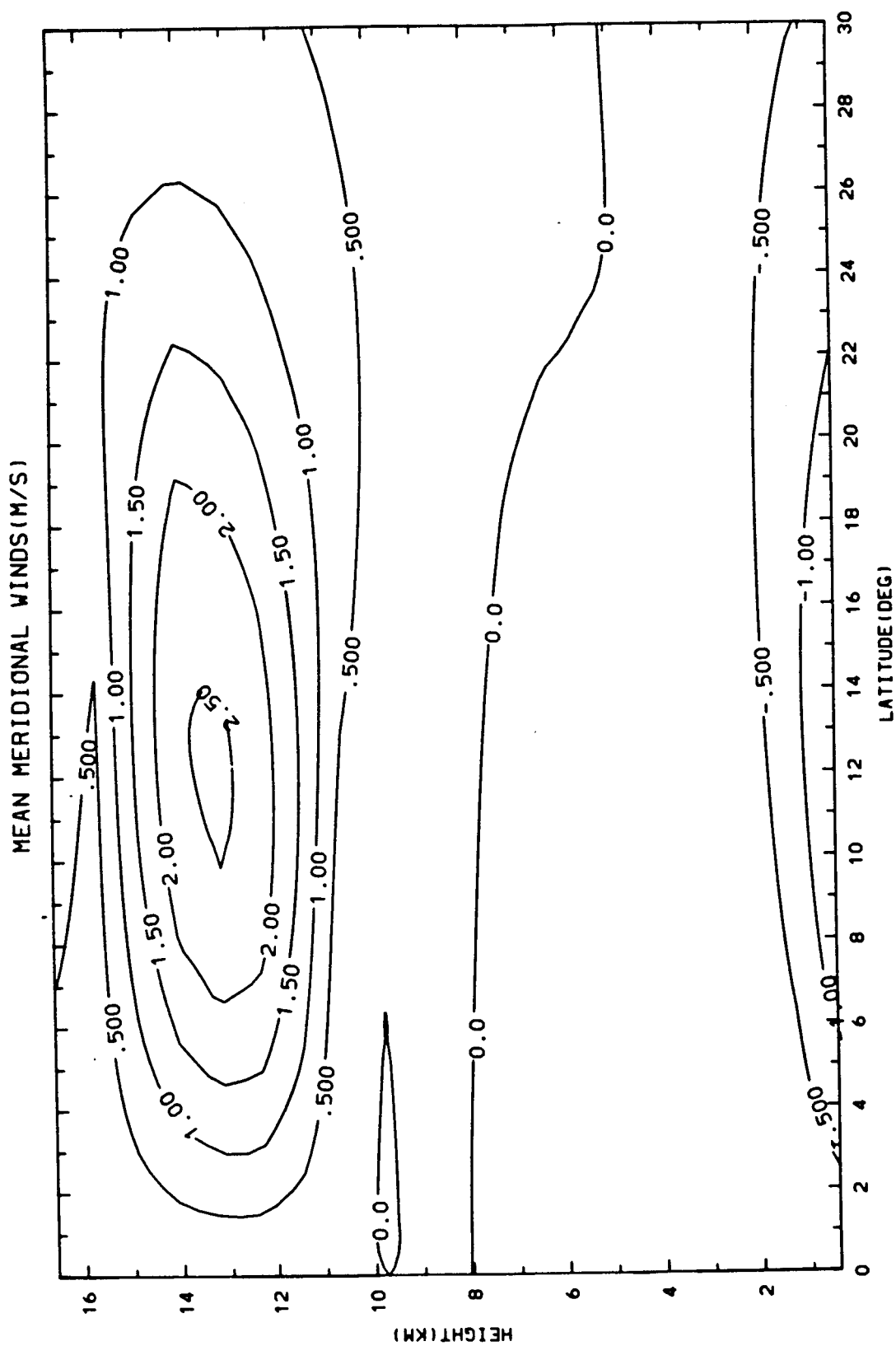


Fig. 5c v-component wind field (m/s) of Hadley cell basic state.

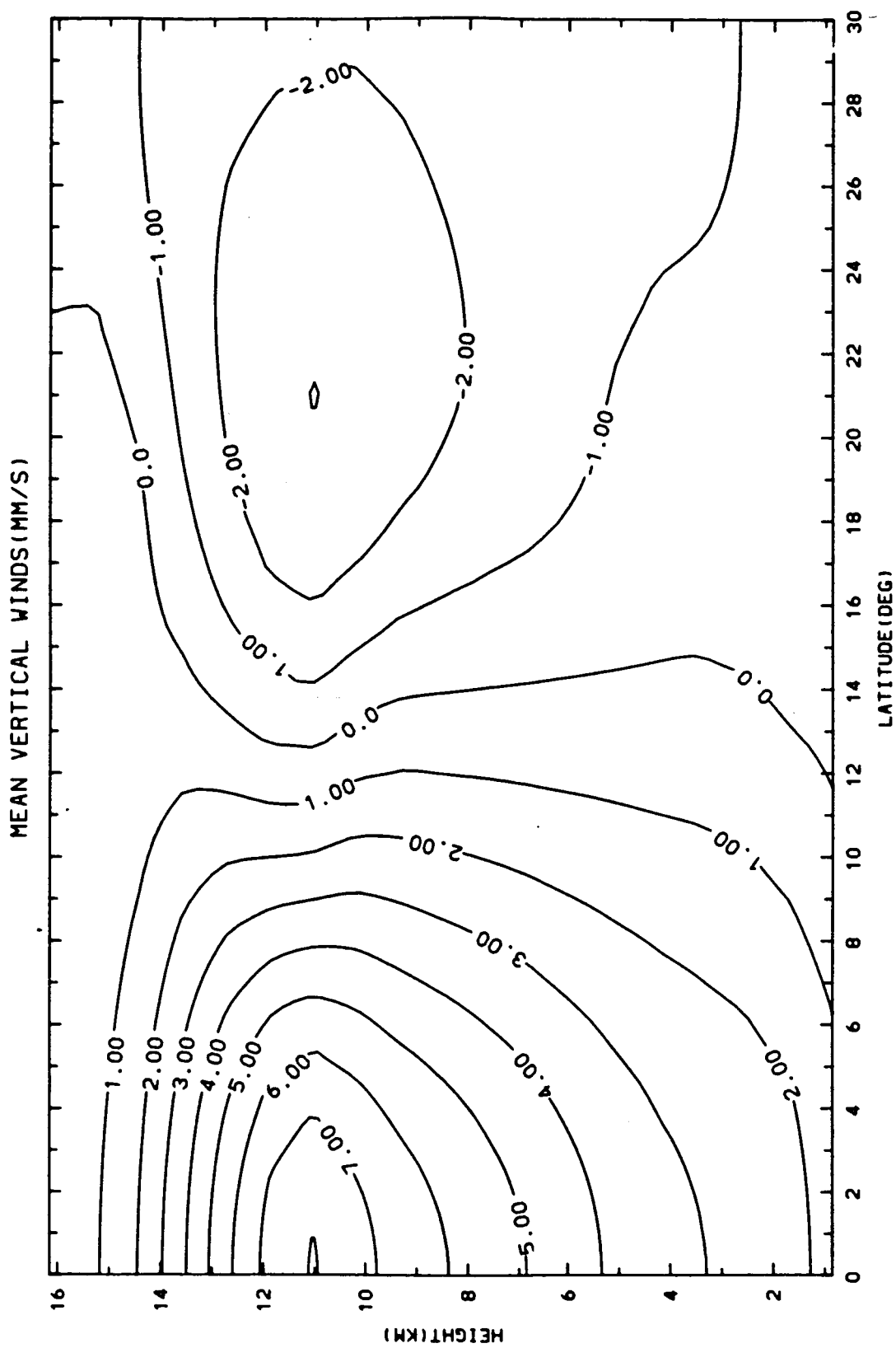


Fig. 5d w-component wind field (mm/s) of Hadley cell basic state.

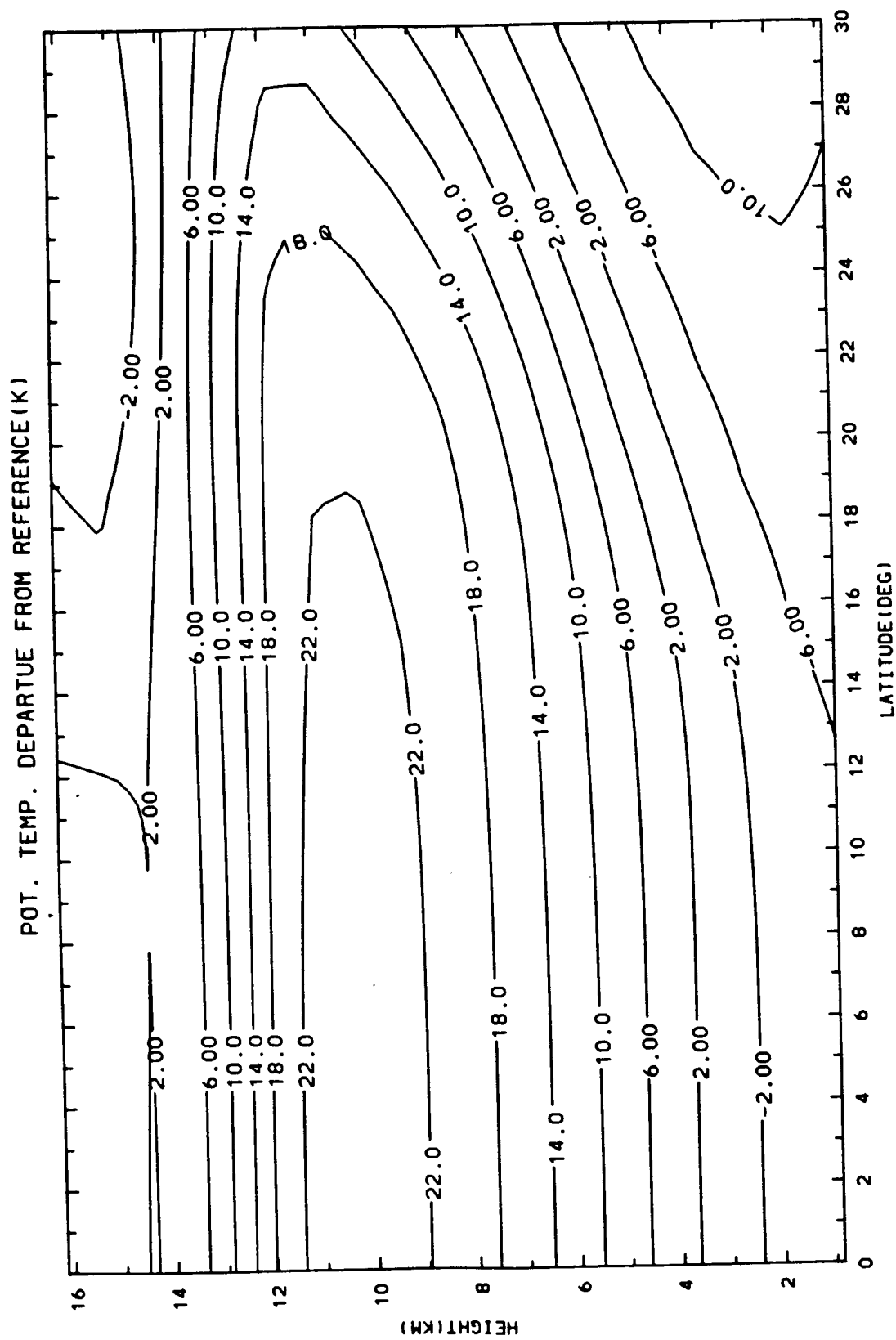


Fig. 5e Potential temperature departure (deg) from reference stratification for Hadley cell basic state.

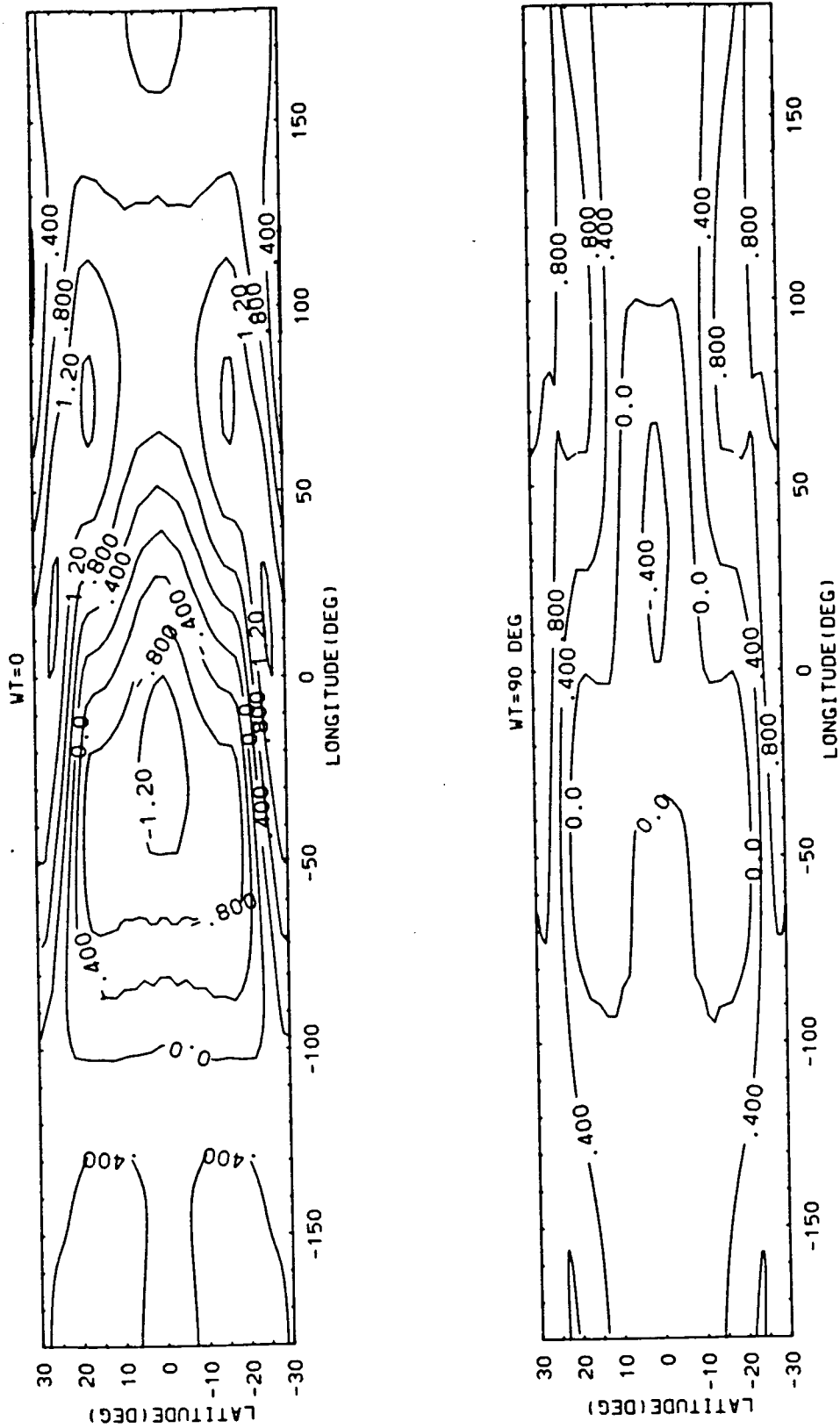


Fig. 6 U-component wind response (m/s) at 200 mb for fully stratified, Hadley cell basic state calculation.

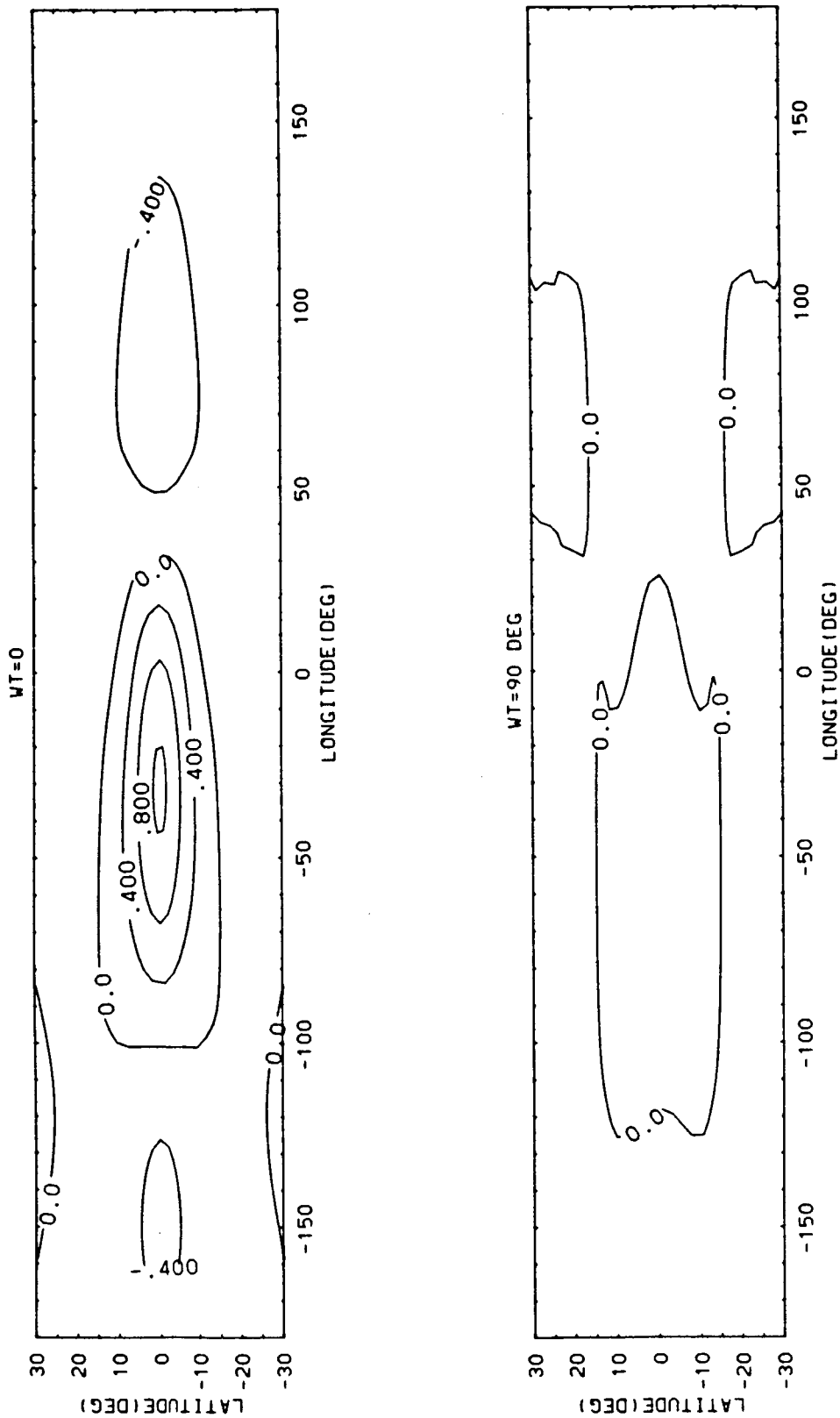


Fig. 7 U-component wind response (m/s) at 830 mb for fully stratified, Hadley cell basic state calculation.

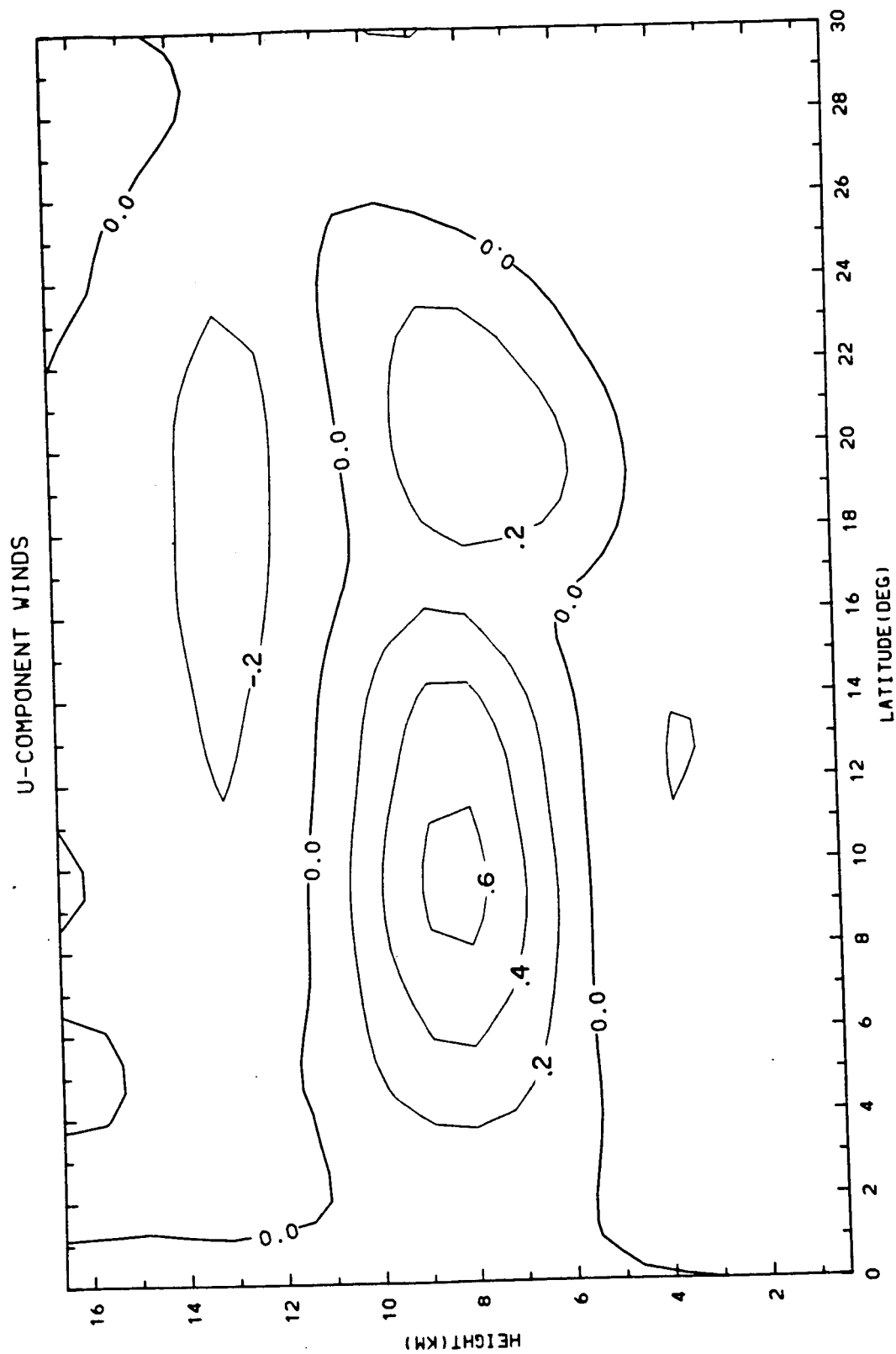


Fig. 8a Wavenumber 0 response at $\omega t = -45$ deg for Hadley cell calculation.

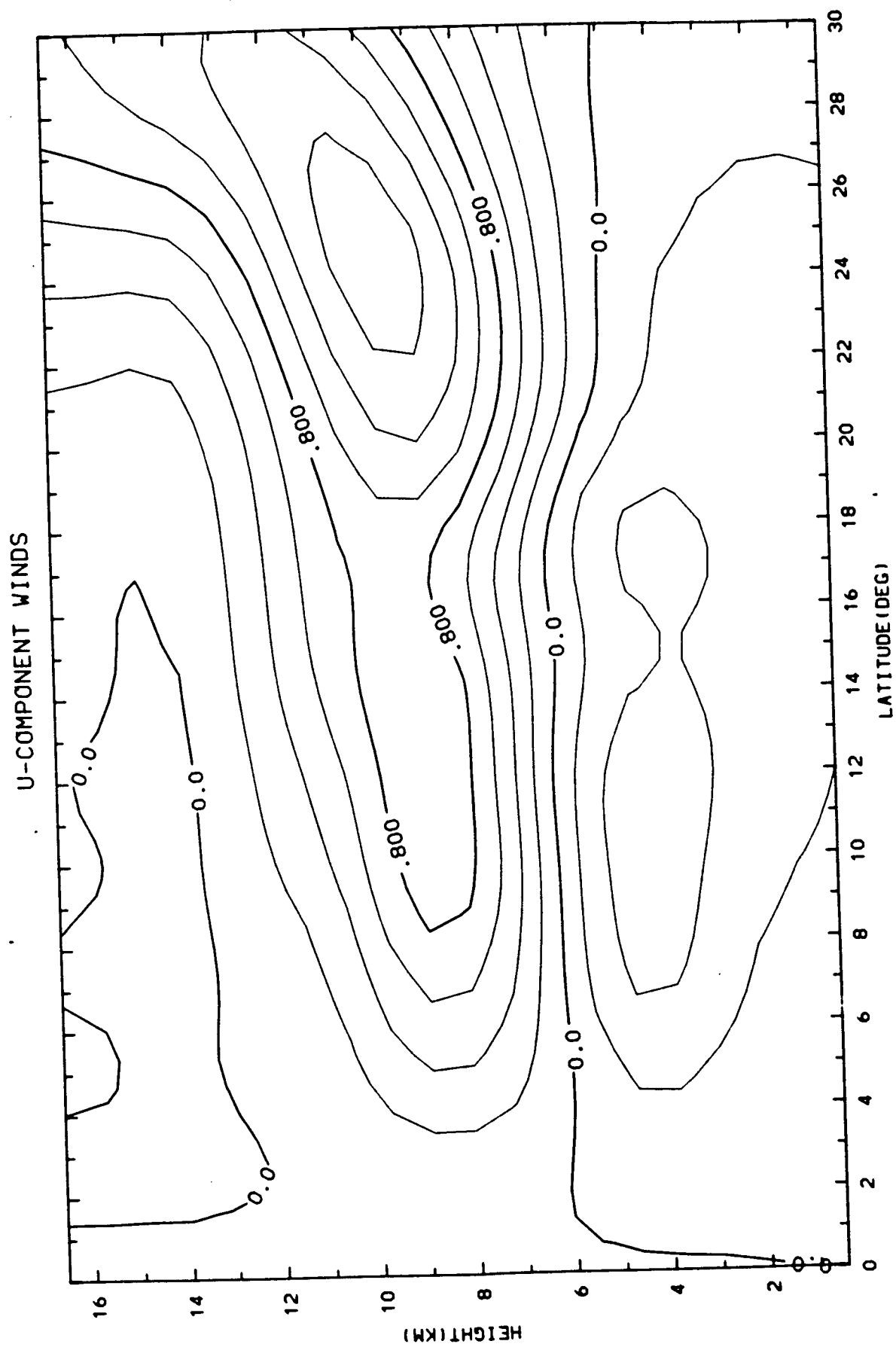


Fig. 8b Wavenumber 0 response at $\omega t = 0$ for Hadley cell calculation.

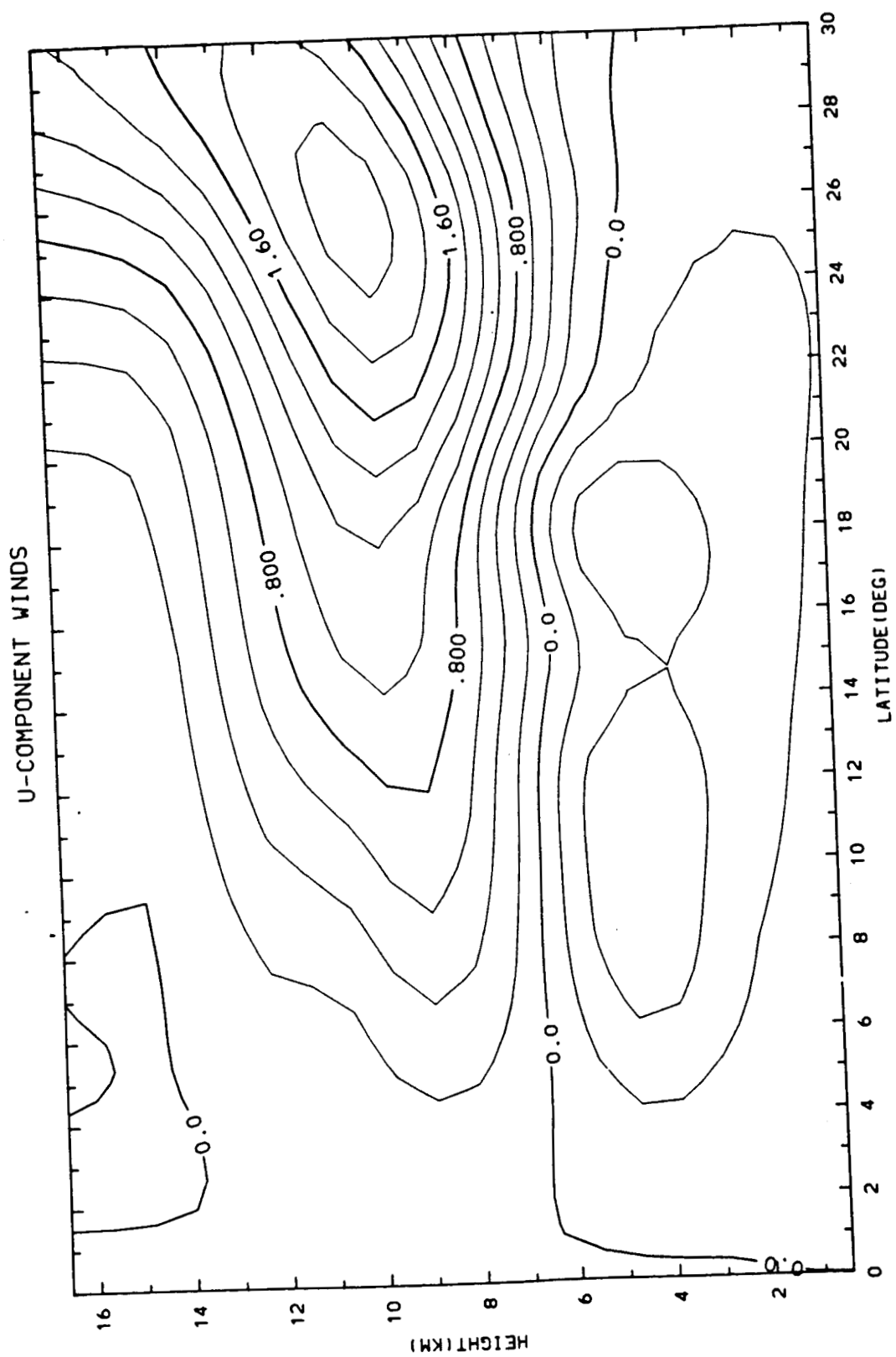


Fig. 8c Wavenumber 0 response at $\omega t = 45$ deg for Hadley cell calculation.

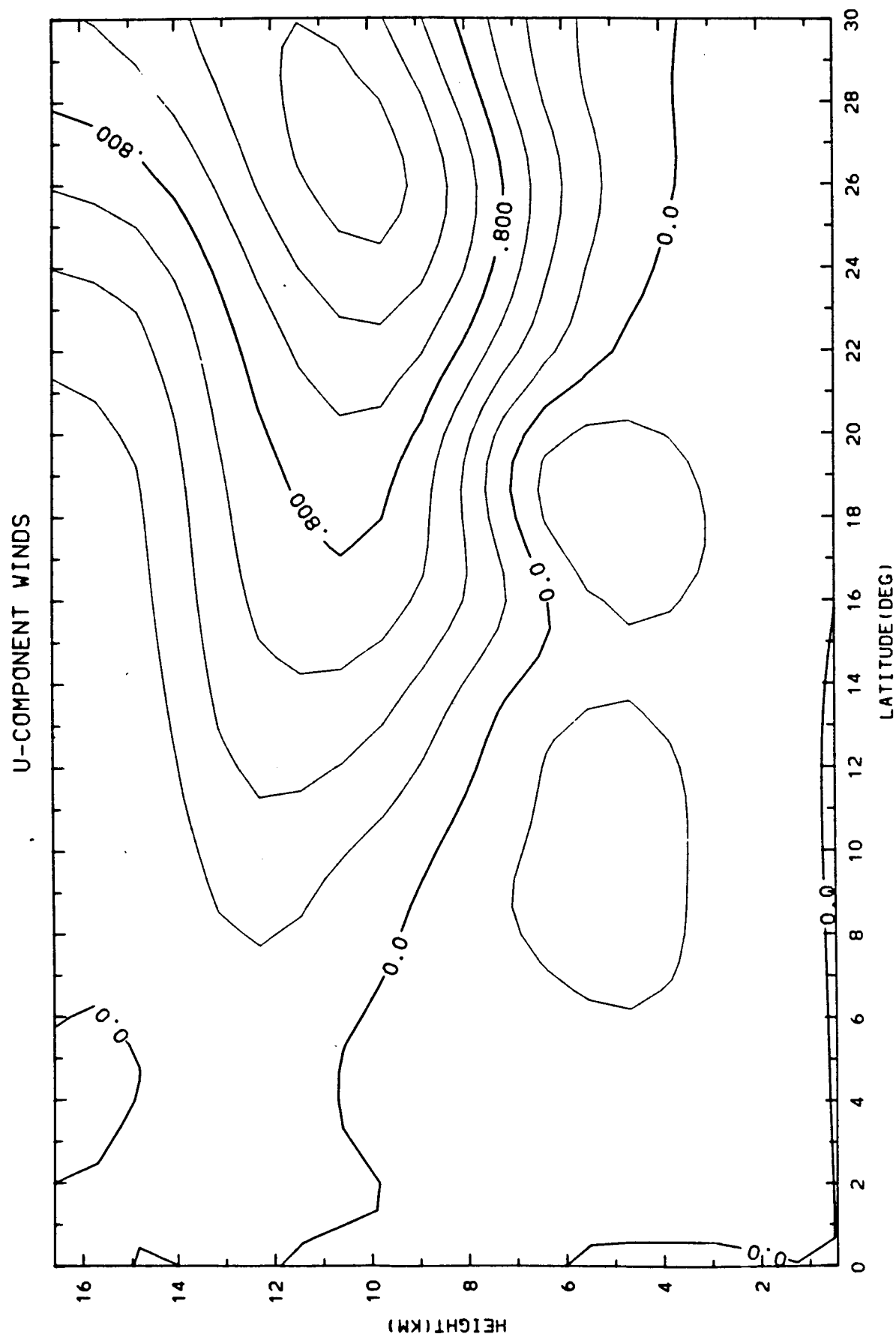


Fig. 8d Wavenumber 0 response at $\omega t = 90$ deg for Hadley cell calculation.

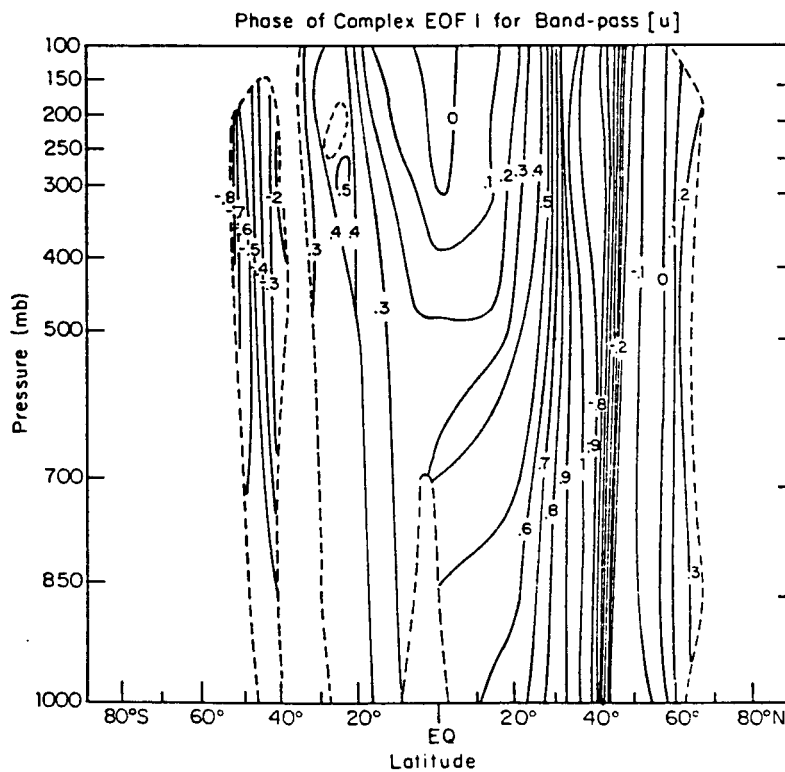
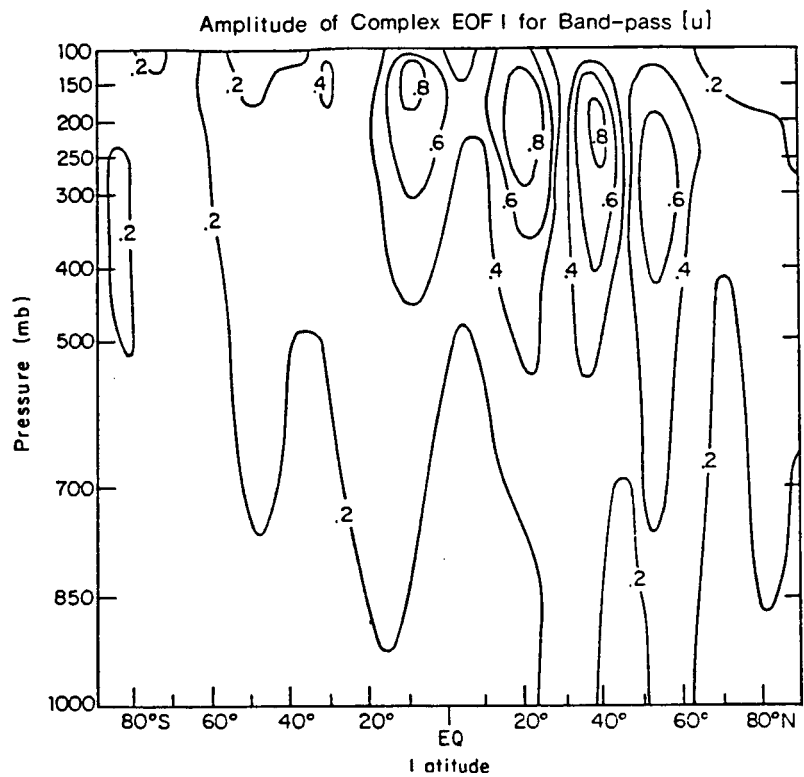


Fig. 9 (a) Amplitude and (b) phase (rad/π) plot of the observed oscillation, from Anderson & Rosen (1983).

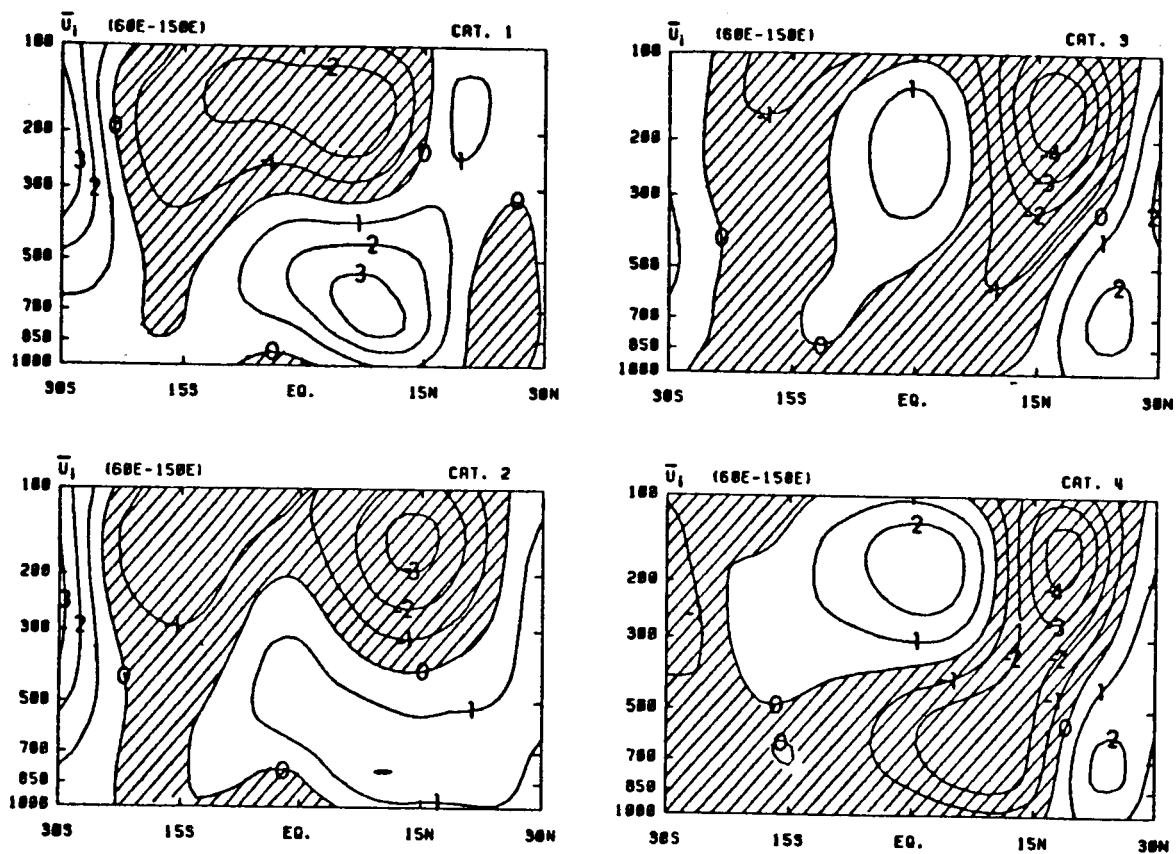


Fig. 10 Latitude-height structure of partial zonal average (60°E to 150°E) u-wind field (with interval 1 ms⁻¹) at 45° phase intervals. From Murakami et al. (1983).